NeurIPS’24 & INFORMS Data Mining Paper Competition Finalist

Cost-Aware Bayesian Optimization with Adaptive Stopping via Gittins Indices

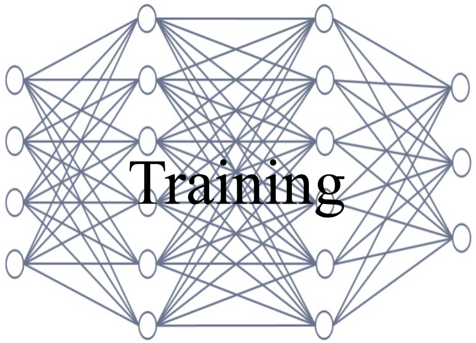
Qian Xie 谢倩 (Cornell ORIE)

Joint work with Linda Cai (UC Berkeley), Theodore Brown (UCL), Raul Astudillo (MBZUAI), Peter Frazier, Alexander Terenin, and Ziv Scully (Cornell)

INFORMS Annual Meeting 2025 Job Market Showcase

Optimization Under Uncertainty

ML model training:

Training hyperparameters →  → Accuracy

(e.g., learning rate, # layers)

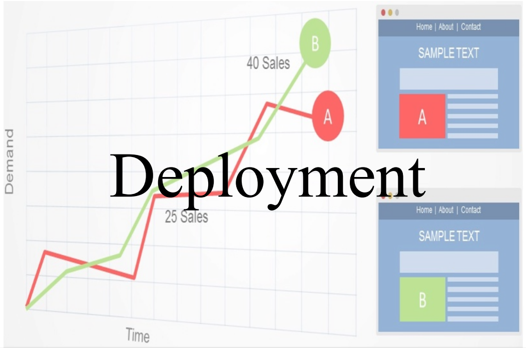
Optimization Under Uncertainty

ML model training:

Training hyperparameters (e.g., learning rate, # layers) ⟶ Training ⟶ Accuracy

Adaptive experimentation:

Decision/design variables (e.g., layout, pricing level) ⟶ Deployment ⟶ Revenue



Optimization Under Uncertainty

Black-box optimization:

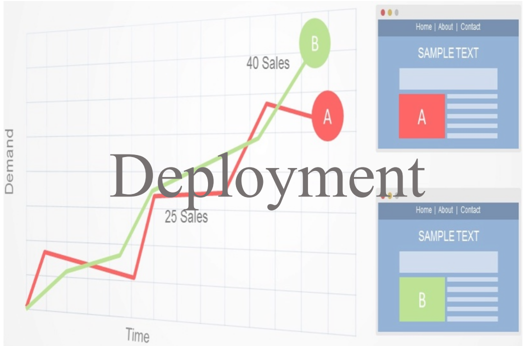
non-analytical & no gradient info

Input x ⟶  ⟶ Performance metric $f(x)$

ML model training:

Training hyperparameters (e.g., learning rate, # layers) ⟶  Training ⟶ Accuracy

Adaptive experimentation:

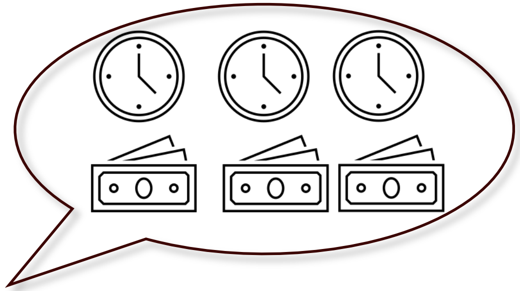
Decision/design variables (e.g., layout, pricing level) ⟶ 

Deployment ⟶ Revenue

Black-Box Optimization

expensive-to-evaluate

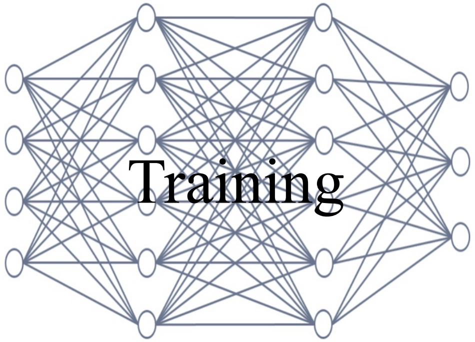
Input x ⟶  ⟶ Performance metric $f(x)$

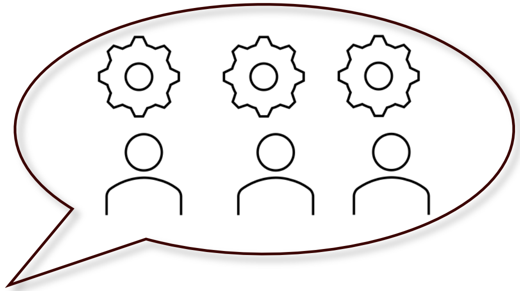


Training time

Compute credits

ML model training:

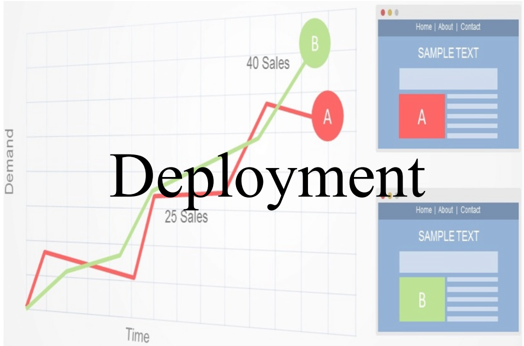
Training hyperparameters (e.g., learning rate, # layers) ⟶  Training ⟶ Accuracy



Operational cost

User experience

Adaptive experimentation:

Decision/design variables (e.g., layout, pricing level) ⟶ 

Deployment ⟶ Revenue

Black-Box Optimization

Input x $\longrightarrow$

expensive-to-evaluate

$\longrightarrow$ Performance metric $f(x)$

High-level goal: Choose $x_1, \ldots, x_T$ to maximize the expected best observed value

$$\mathbb{E} \max_{t=1,2,\ldots,T} f(x_t)$$

Data-Driven Black-Box Optimization

expensive-to-evaluate

Input x $\longrightarrow$  $\longrightarrow$ Performance metric $f(x)$

adaptively

High-level goal: Choose $x_1, \ldots, x_T$ to maximize the expected best observed value

$$\mathbb{E} \max_{t=1,2,\ldots,T} f(x_t)$$

Fewer #evaluations

Data-Driven Black-Box Optimization

Input x $\longrightarrow$  $\longrightarrow$ Performance metric $f(x)$

expensive-to-evaluate

adaptively

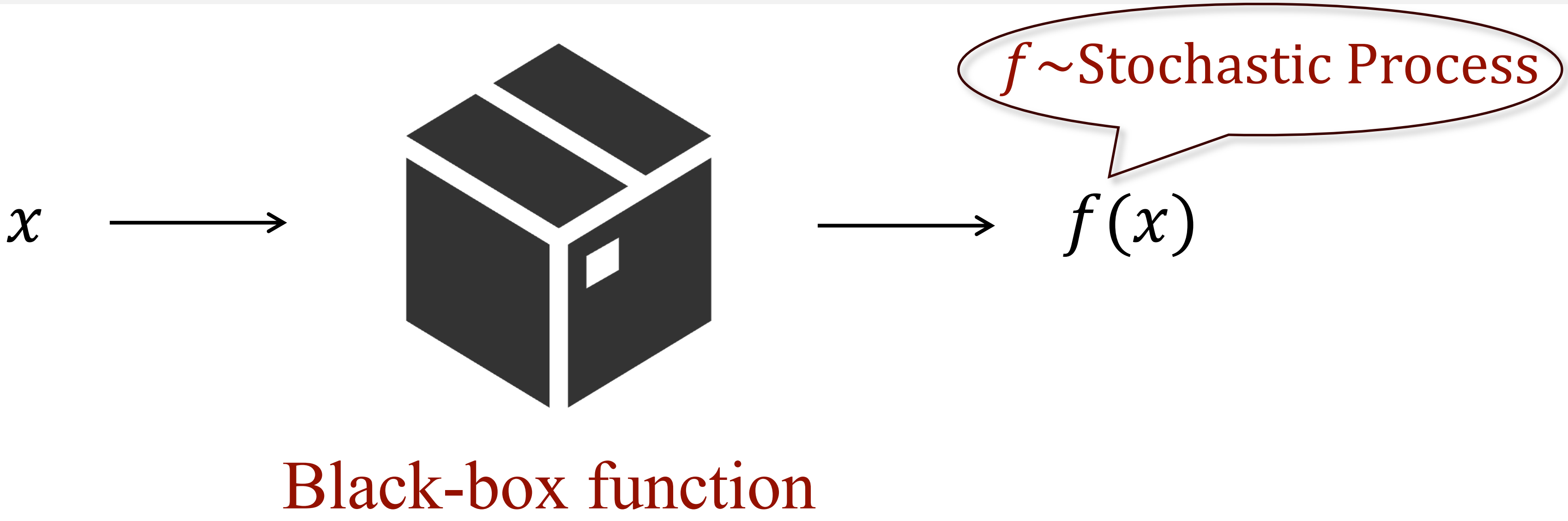
High-level goal: Choose $x_1, \ldots, x_T$ to maximize the expected best observed value

$$\mathbb{E} \max_{t=1,2,\ldots,T} f(x_t)$$

Fewer #evaluations

Efficient framework: Bayesian optimization

Bayesian Optimization



Bayesian Optimization

Time 0

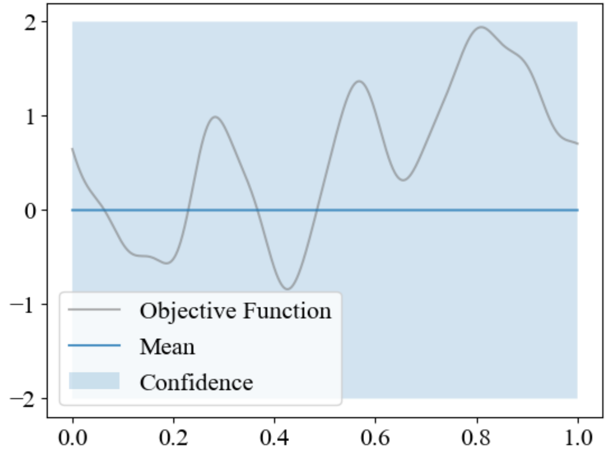
$f \sim$Stochastic Process

x

$f(x)$

Model belief

Black-box function



Probabilistic model

(e.g., Gaussian process)

Bayesian Optimization

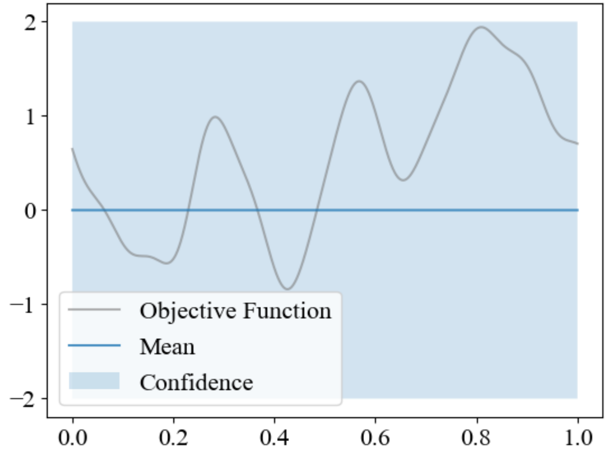
Time t

$x_1, \dots, x_t$ →  → $f(x_1), \dots, f(x_t)$

Black-box function

$f \sim$Stochastic Process

Model belief



Probabilistic model

(e.g., Gaussian process)

Bayesian Optimization

Time t

$x_1, \dots, x_t$

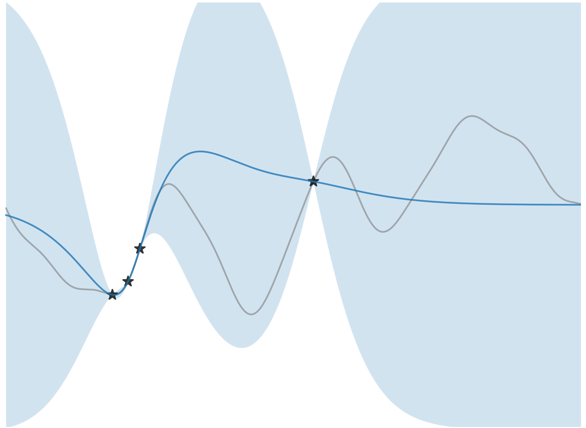
Black-box function

$f \sim$ Stochastic Process

$f(x_1), \dots, f(x_t)$

Update belief

(Bayes' rule)



Probabilistic model

(e.g., Gaussian process)

Bayesian Optimization

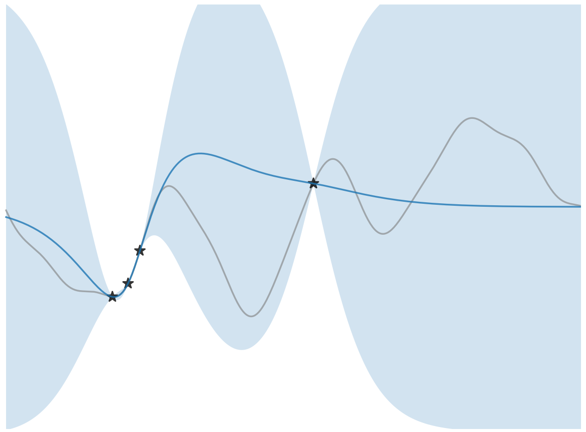
Time t

$x_1, \ldots, x_t$ →  → $f(x_1), \ldots, f(x_t)$

Black-box function

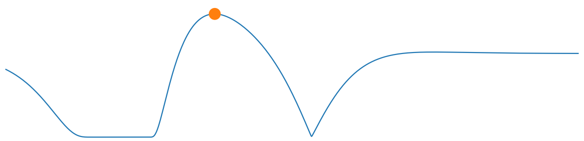
Update belief

(Bayes' rule)



Probabilistic model

(e.g., Gaussian process)



Acquisition function

(e.g., EI, UCB, TS)

scoring (worth of each point

Bayesian Optimization

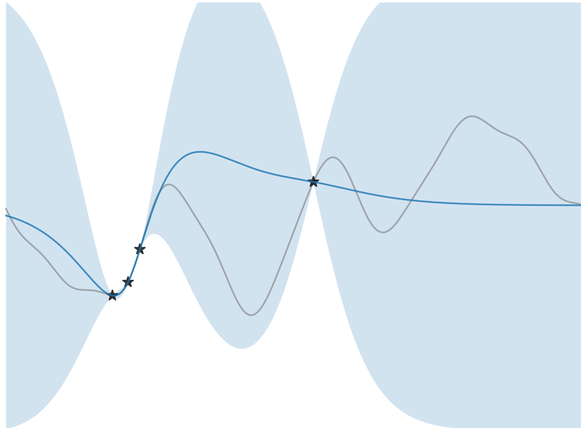
Time t

$x_1, \dots, x_t$ $\longrightarrow$

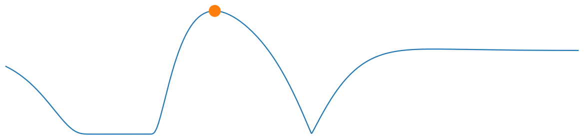
Black-box function

$\longrightarrow$ $f(x_1), \dots, f(x_t)$

Update belief
(Bayes' rule)



Probabilistic model
(e.g., Gaussian process)



Acquisition function
(e.g., EI, UCB, TS)

scoring (worth of each point

Acquire next input
(gradient-based optimization)

Bayesian Optimization

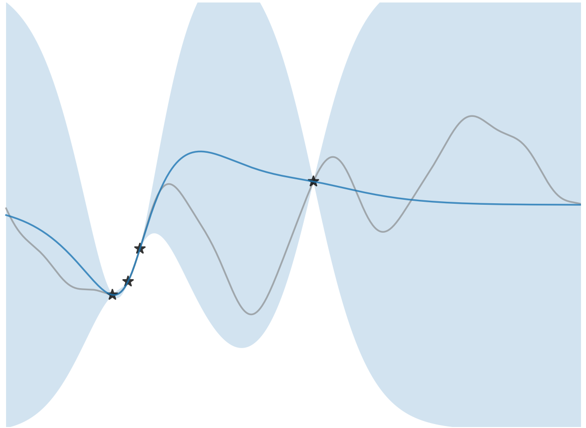
Time $t + 1$

$x_1, \dots, x_{t+1}$ $\longrightarrow$

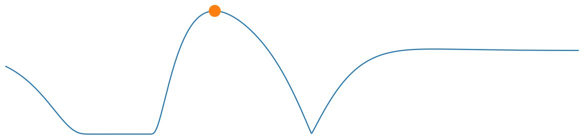
Black-box function

$\longrightarrow$ $f(x_1), \dots, f(x_{t+1})$

Update belief
(Bayes' rule)



Probabilistic model
(e.g., Gaussian process)

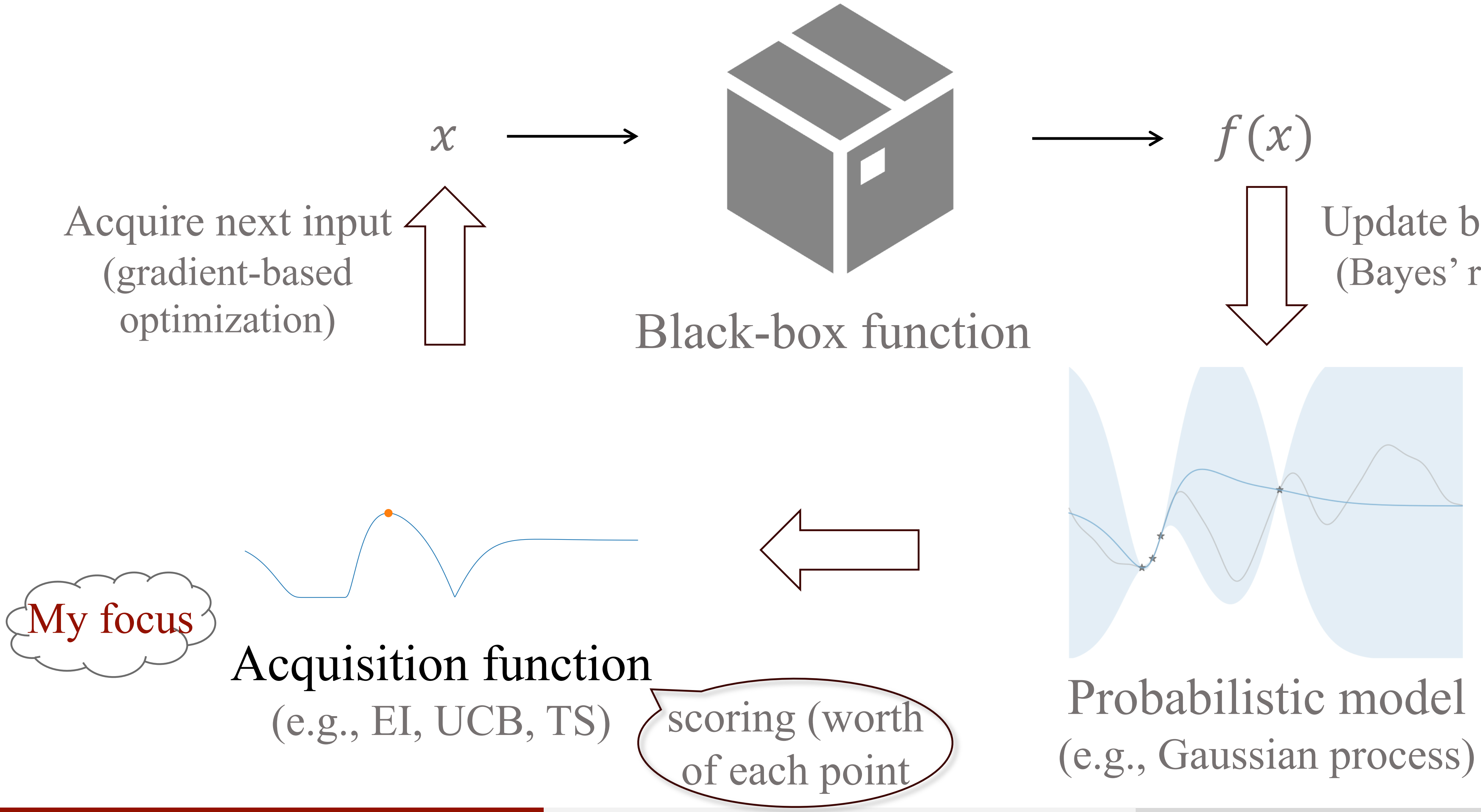


Acquisition function
(e.g., EI, UCB, TS)

scoring (worth of each point

Acquire next input
(gradient-based optimization)

Bayesian Optimization



Existing Design Principles

- Improvement-based (e.g., EI)
- Entropy-based
- Confidence bounds (UCB/LCB)
- Thompson sampling (TS)

New Design Principle: Gittins Index

- Improvement-based (e.g., EI)
- Entropy-based
- Confidence bounds (UCB/LCB)
- Thompson sampling (TS)
- Gittins Index

New Design Principle: Gittins Index

- Improvement-based (e.g., EI)
- Entropy-based
- Confidence bounds (UCB/LCB)
- Thompson sampling (TS)
- Gittins Index

? Why another principle?

Our Contribution: Gittins Index Principle

- Improvement-based (e.g., EI)
- Entropy-based
- Confidence bounds (UCB/LCB)
- Thompson sampling (TS)
- Gittins Index

? Why another principle?

1. Naturally incorporates side info and practical flexibility
2. Performs competitively on benchmarks
3. Comes with theoretical guarantees

Our Contribution: Gittins Index Principle

- Improvement-based (e.g., EI)
- Entropy-based
- Confidence bounds (UCB/LCB)
- Thompson sampling (TS)
- Gittins Index

? Why another principle?

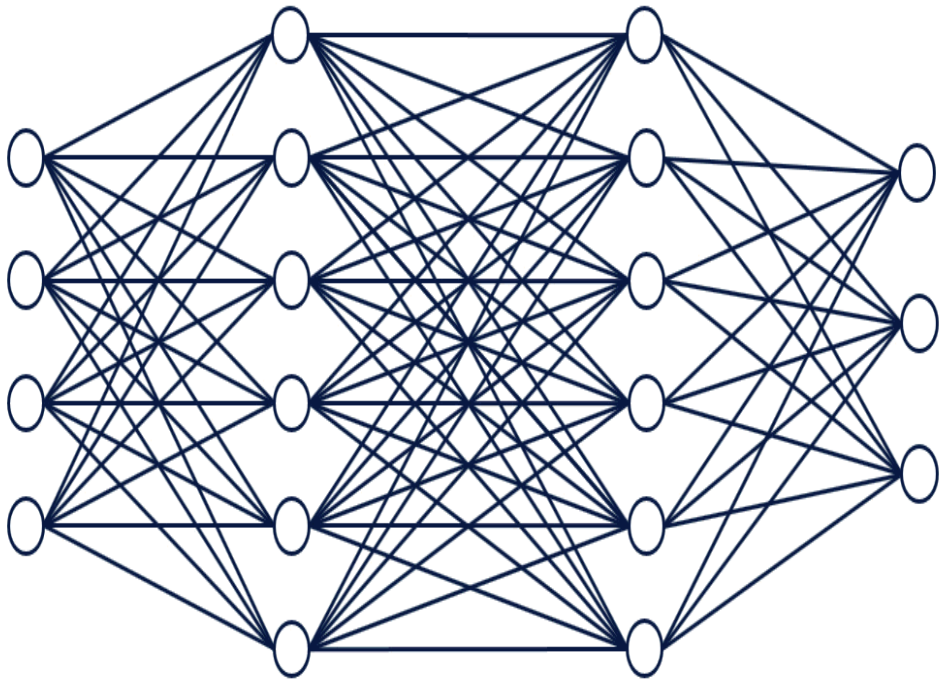
1. Naturally incorporates side info and practical flexibility
2. Performs competitively on benchmarks
3. Comes with theoretical guarantees

Under-explored Side Info and Flexibility

Varying evaluation costs

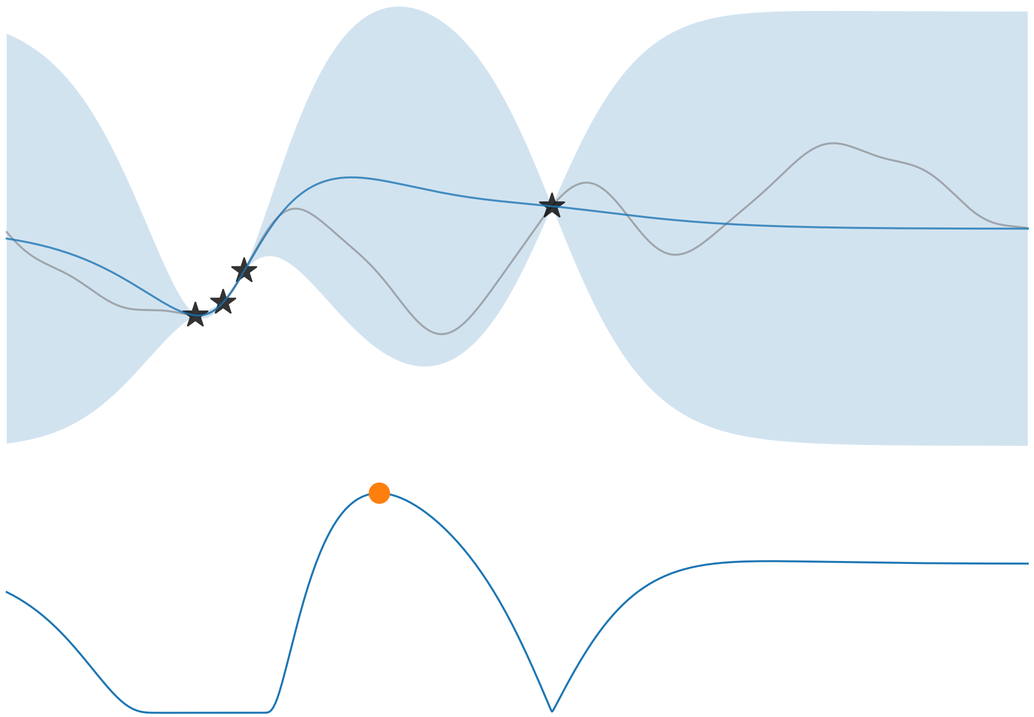
Smart stopping time

Observable multi-stage feedback



How does existing principle incorporate them?

- Varying evaluation costs
- Smart stopping time
- Observable multi-stage feedback



Expected improvement $\mathrm{EI}(x)$

How does existing principle incorporate them?

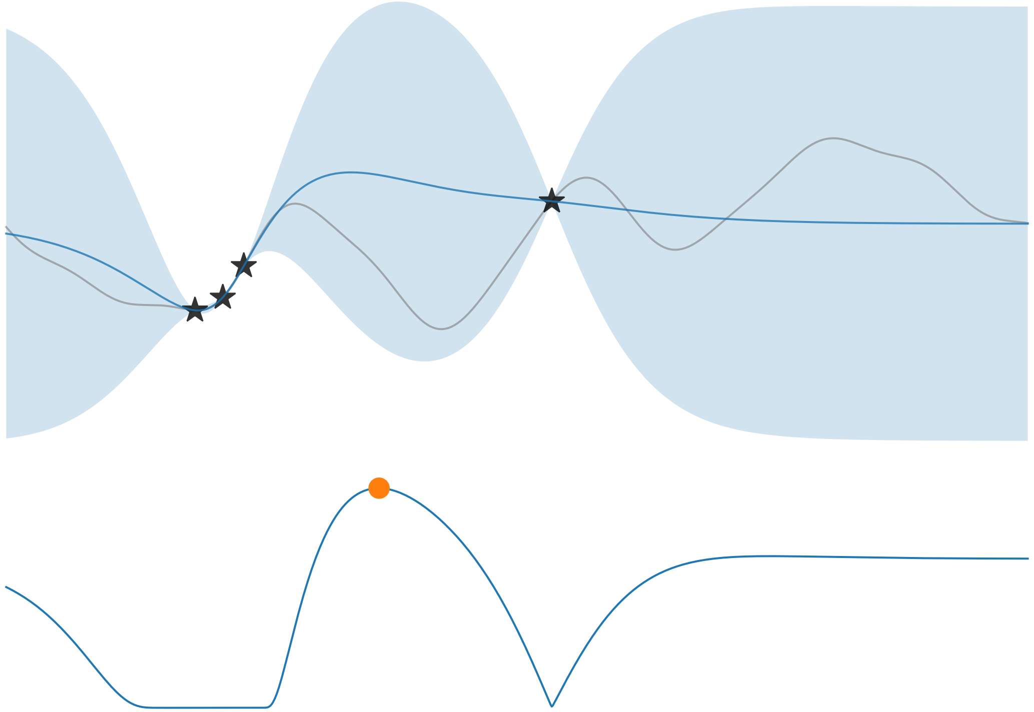
Varying evaluation costs

$\mathrm{EI}(x)/c(x)$

Why divide?

Smart stopping time

Observable multi-stage feedback



Expected improvement $\mathrm{EI}(x)$

How does existing principle incorporate them?

Varying evaluation costs

$\mathrm{EI}(x)/c(x)$

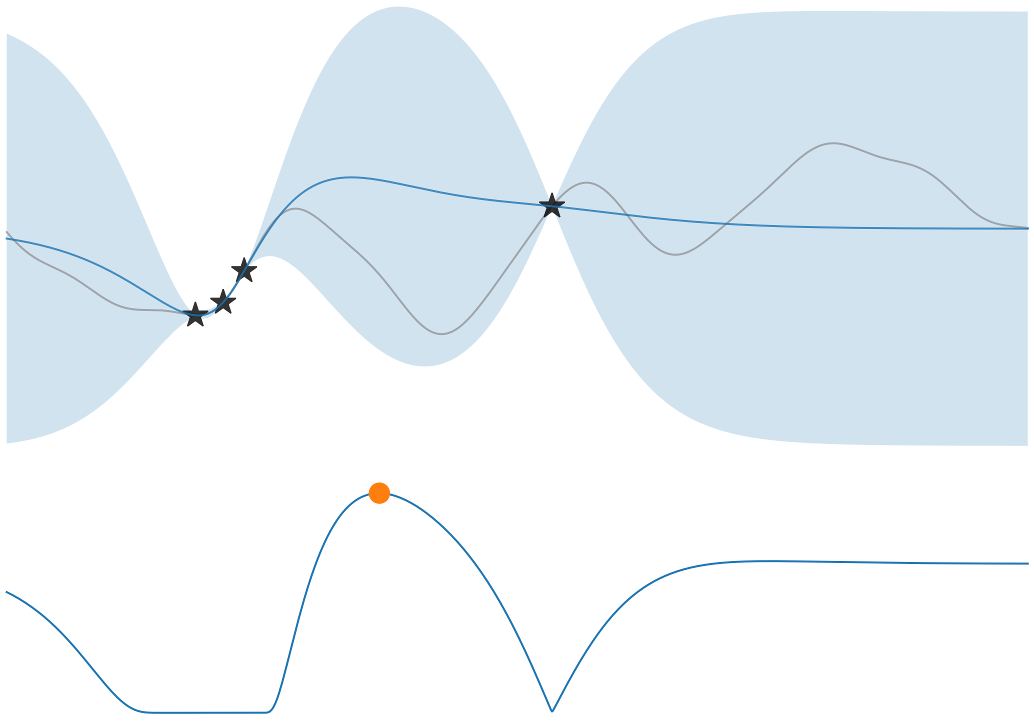
Why divide?

Smart stopping time

$\mathrm{EI}(x) \leq \theta$

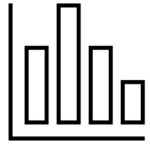
Which threshold?

Observable multi-stage feedback



Expected improvement $\mathrm{EI}(x)$

How does existing principle incorporate them?

 Varying evaluation costs

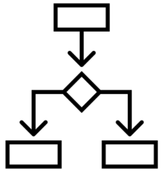
$\mathrm{EI}(x)/c(x)$

Why divide?

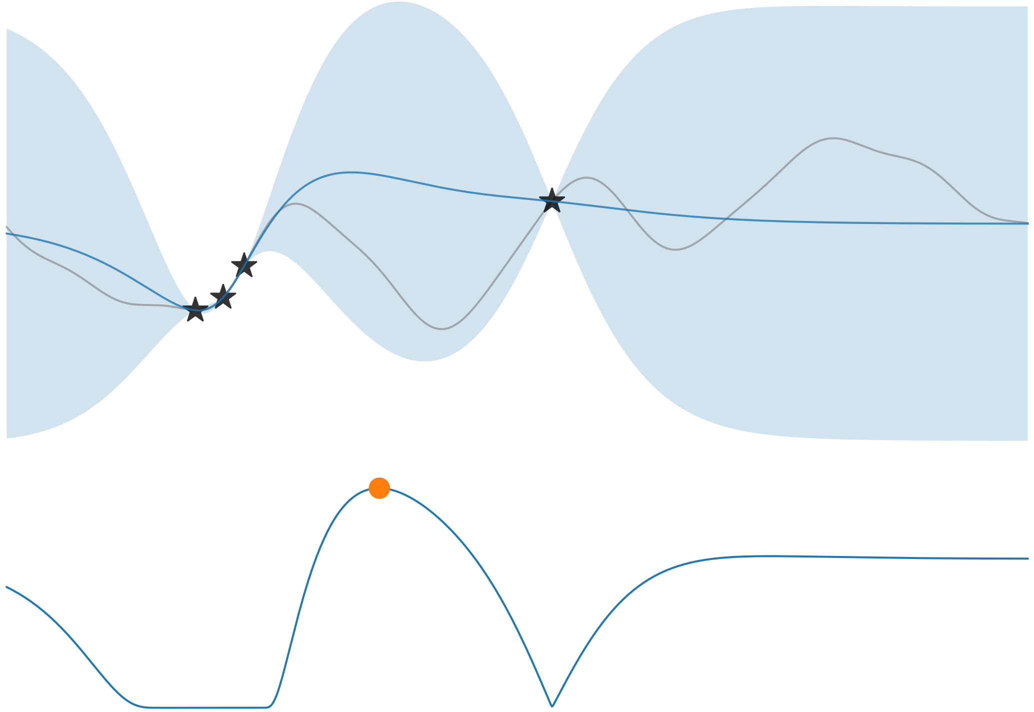
 Smart stopping time

$\mathrm{EI}(x) \leq \theta$

Which threshold?

 Observable multi-stage feedback

?



Expected improvement $\mathrm{EI}(x)$

Under-explored Side Info and Flexibility

Varying evaluation costs

Smart stopping time

Observable multi-stage feedback

New design principle:
Gittins index

Why Gittins index?

Varying evaluation costs

Smart stopping time

Observable multi-stage feedback

New design principle:
Gittins index

Why Gittins index?

Varying evaluation costs

Smart stopping time

Observable multi-stage feedback

New design principle: Gittins index

Optimal in related sequential decision problems

Why Gittins index?

Varying evaluation costs

Features in Pandora's box

Smart stopping time

Features in Pandora's box

Observable multi-stage feedback

New design principle:
Gittins index

Optimal in related sequential decision problems

Why Gittins index?

Varying evaluation costs

Features in Pandora's box

Smart stopping time

Features in Pandora's box

Observable multi-stage feedback

Features in Markovian bandits

New design principle:
Gittins index

Optimal in related sequential decision problems

What is Pandora's Box?

Varying evaluation costs

Features in Pandora's box

Smart stopping time

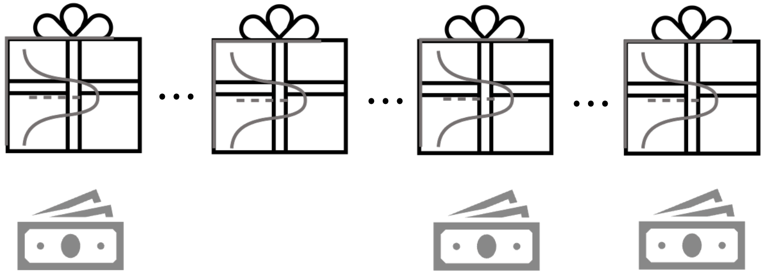
Features in Pandora's box

Observable multi-stage feedback

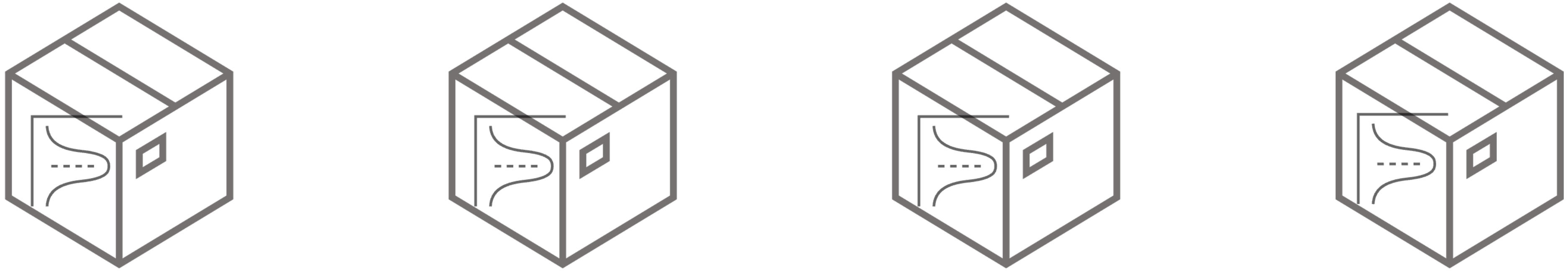
Features in Markovian bandits

New design principle:
Gittins index

Optimal in related sequential decision problems



Pandora's Box



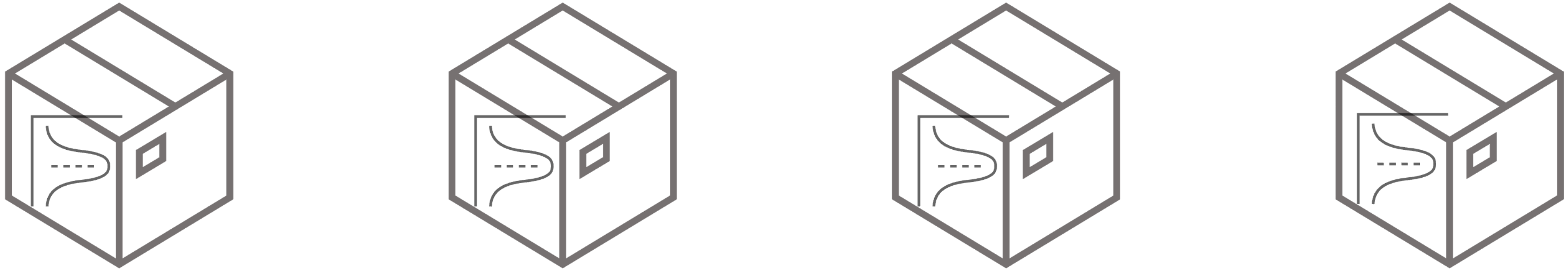
High-level goal: Choose box $x_1, \ldots, x_T$ to open to maximize the expected utility

$$\mathbb{E} \max_{t=1,2,\ldots,T} f(x_t) - \mathbb{E} \sum_{t=1}^{T} c(x_t)$$

Flexible stopping time

Pandora’s Box

$t = 0$

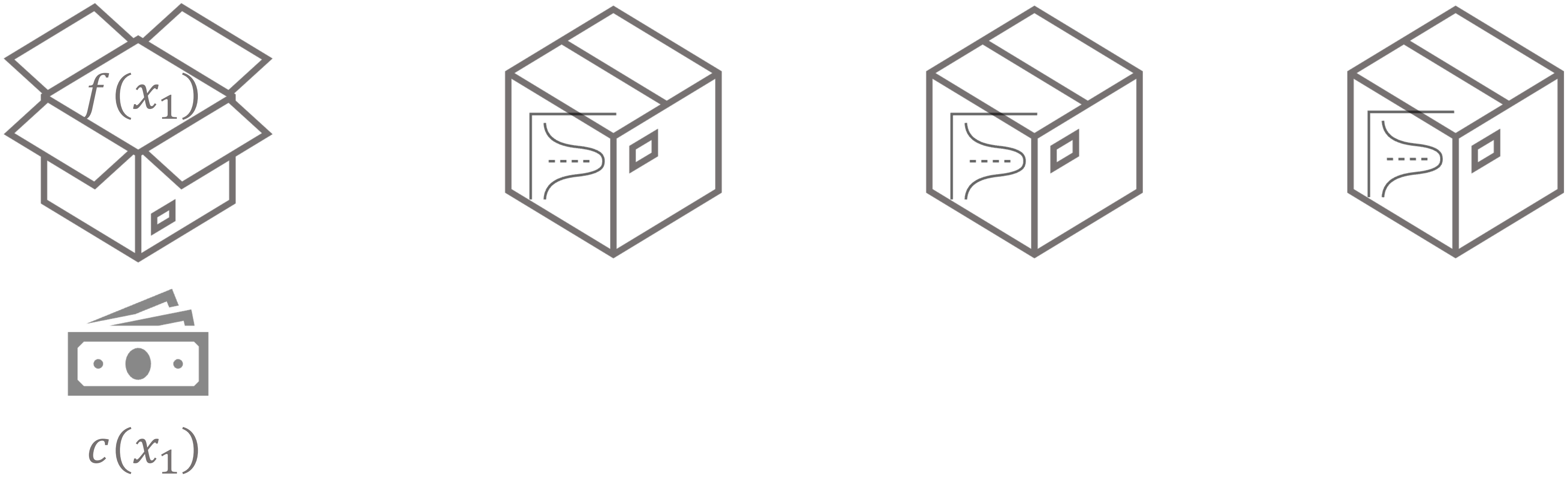


High-level goal: Choose box $x_1, \ldots, x_T$ to open to maximize the expected utility

$$\mathbb{E} \max_{t=1,2,\ldots,T} f(x_t) - \mathbb{E} \sum_{t=1}^{T} c(x_t)$$

Pandora's Box

$t = 1$

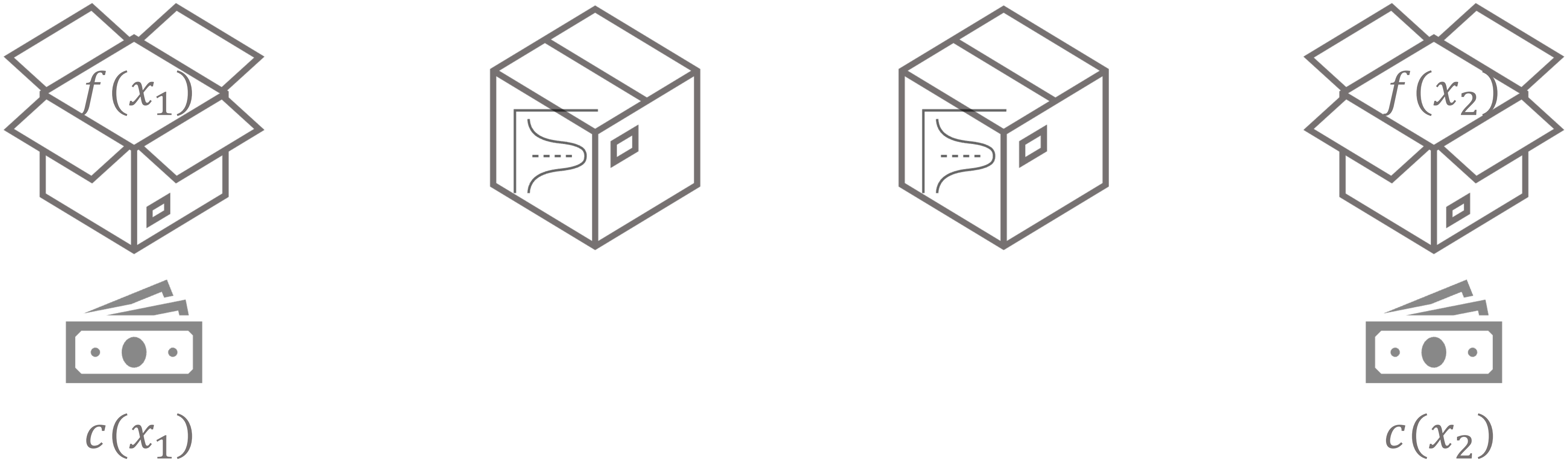


High-level goal: Choose box $x_1, \ldots, x_T$ to open to maximize the expected utility

$$\mathbb{E} \max_{t=1,2,\ldots,T} f(x_t) - \mathbb{E} \sum_{t=1}^{T} c(x_t)$$

Pandora's Box

$t = 2$

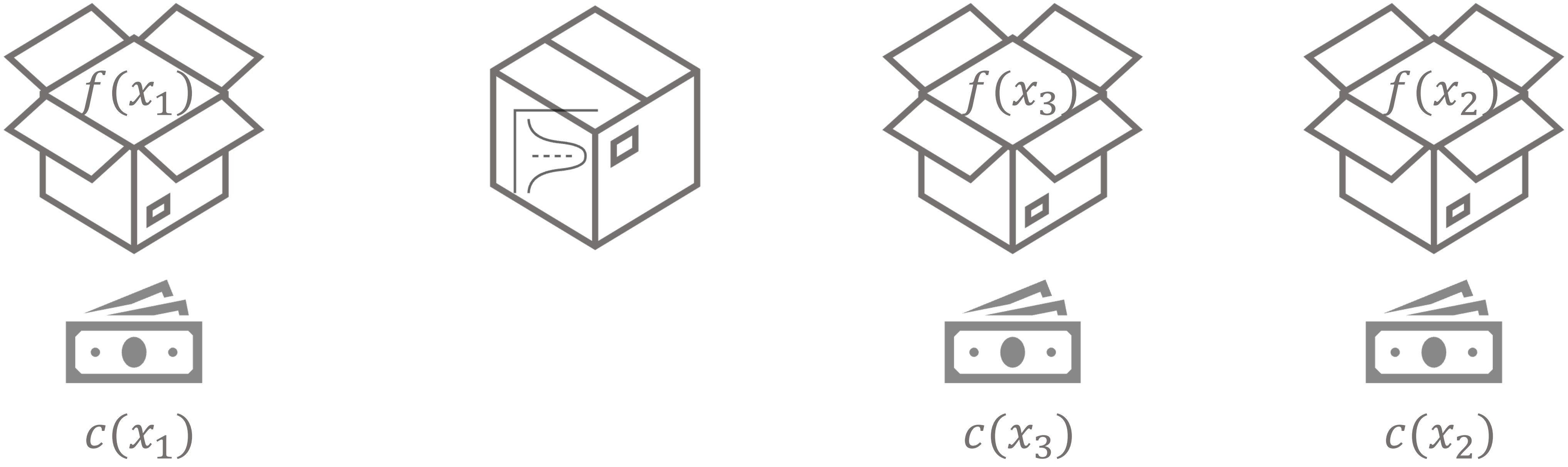


High-level goal: Choose box $x_1, \ldots, x_T$ to open to maximize the expected utility

$$\mathbb{E} \max_{t=1,2,\ldots,T} f(x_t) - \mathbb{E} \sum_{t=1}^{T} c(x_t)$$

Pandora's Box

$t = 3$

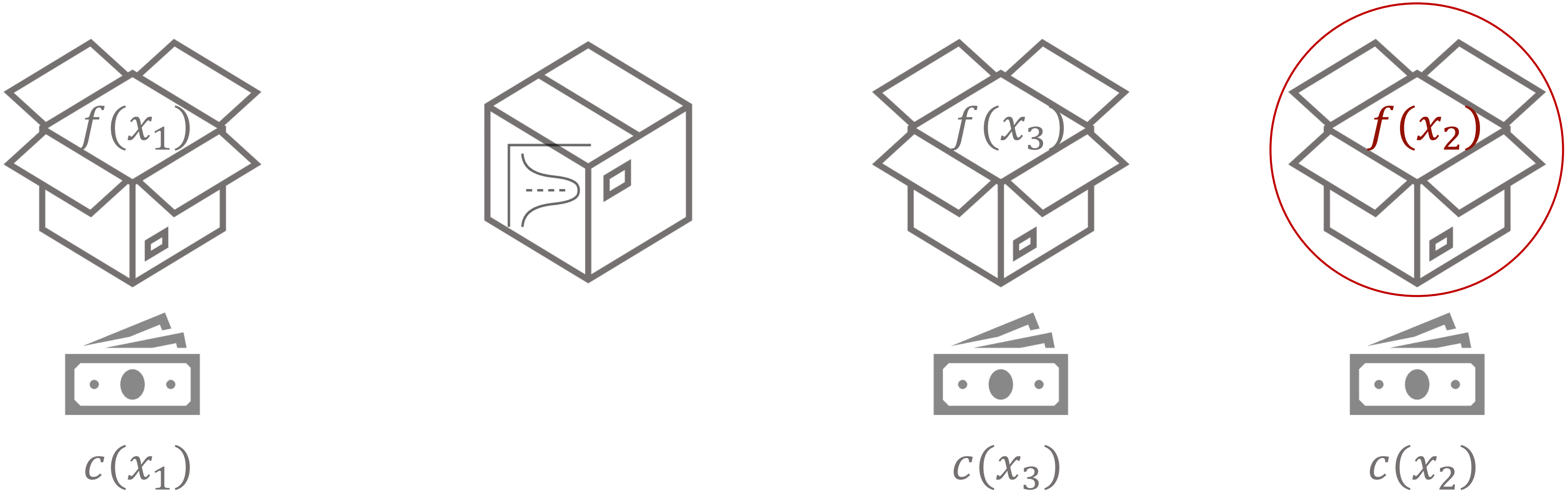


High-level goal: Choose box $x_1, \dots, x_T$ to open to maximize the expected utility

$$\mathbb{E} \max_{t=1,2,\dots,T} f(x_t) - \mathbb{E} \sum_{t=1}^{T} c(x_t)$$

Pandora's Box

$t = T$, stop

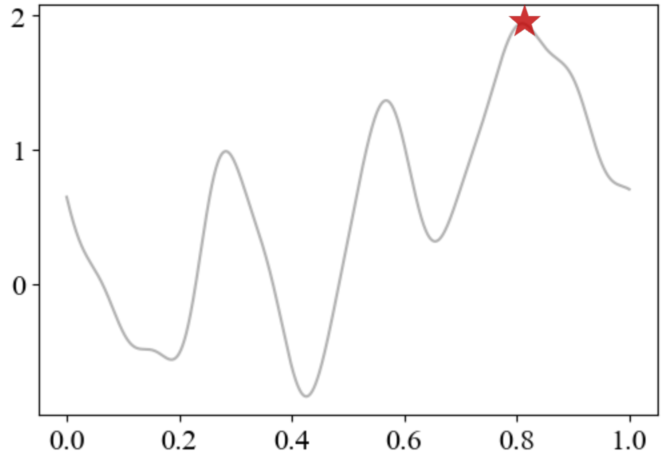


High-level goal: Choose box $x_1, \ldots, x_T$ to open to maximize the expected utility

$$\mathbb{E} \max_{t=1,2,\ldots,T} f(x_t) - \mathbb{E} \sum_{t=1}^{T} c(x_t)$$

Bayesian Optimization

$f(x)$



Continuous

Correlated

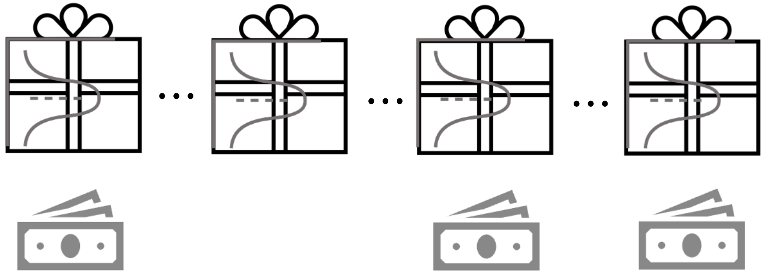
Fixed-iteration

Expected best-observed value

$$\mathbb{E} \max_{t=1,2,\ldots,T} f(x_t)$$

Pandora's Box

[Weitzman'79]



$f(x)$

$c(x)$

Discrete

Independent

Flexible-stopping

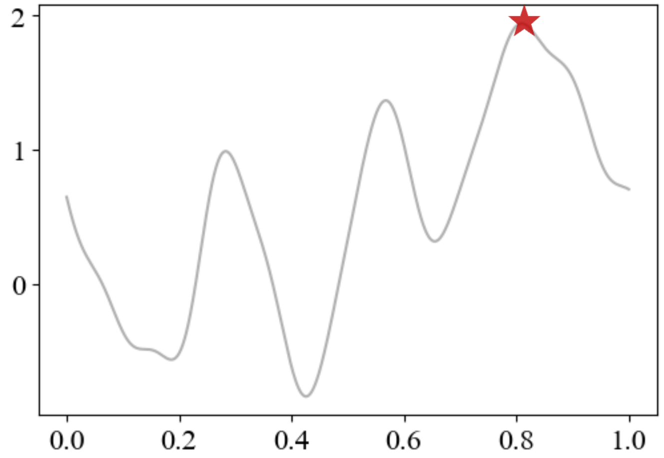
Expected utility

cumulative cost

$$\mathbb{E} \max_{t=1,2,\ldots,T} f(x_t) - \mathbb{E} \sum_{t=1}^{T} c(x_t)$$

Bayesian Optimization

$f(x)$



Continuous

Correlated

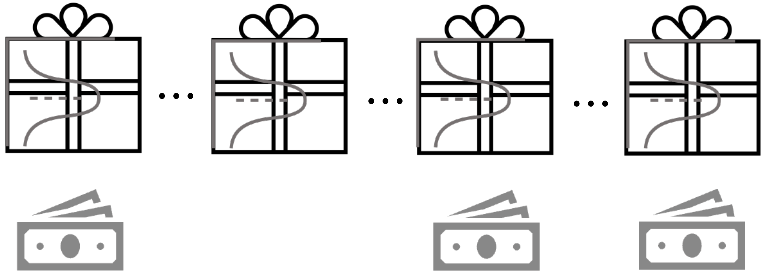
Fixed-iteration

Expected best-observed value

$$\mathbb{E} \max_{t=1,2,\ldots,T} f(x_t)$$

Pandora's Box

[Weitzman'79]



$f(x)$

$c(x)$

Discrete

Independent

Flexible-stopping

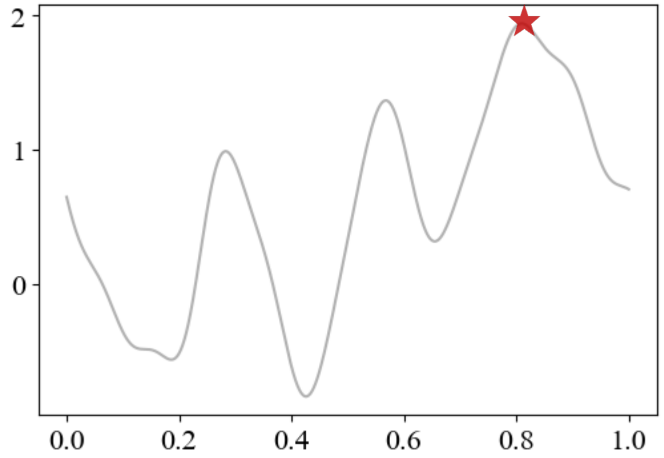
Expected utility

cumulative cost

$$\mathbb{E} \max_{t=1,2,\ldots,T} f(x_t) - \mathbb{E} \sum_{t=1}^{T} c(x_t)$$

Bayesian Optimization

$f(x)$



Continuous

Correlated

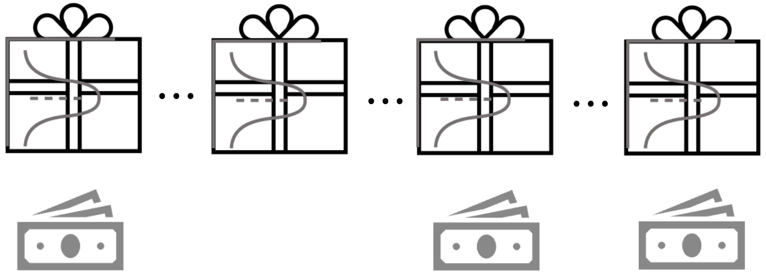
Fixed-iteration

Expected regret

$$\mathbb{E} \max_{x \in \mathcal{X}} f(x) - \mathbb{E} \max_{t=1,2,\ldots,T} f(x_t)$$

Pandora's Box

[Weitzman'79]



$f(x)$

$c(x)$

Discrete

Independent

Flexible-stopping

Expected utility

cumulative cost

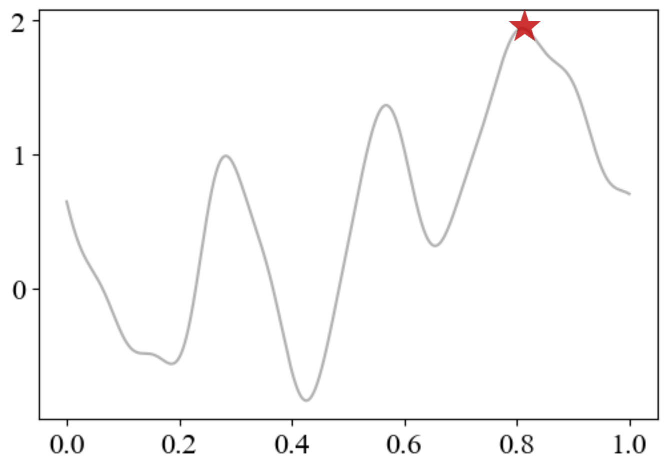
$$\mathbb{E} \max_{t=1,2,\ldots,T} f(x_t) - \mathbb{E} \sum_{t=1}^{T} c(x_t)$$

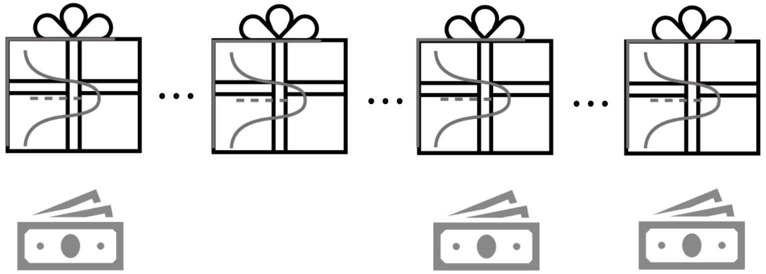
Bayesian Optimization

Pandora's Box

[Weitzman'79]

$f(x)$





$f(x)$

$c(x)$

Bayesian Optimization	Pandora's Box
Continuous	Discrete
Correlated	Independent
Fixed-iteration	Flexible-stopping

Expected regret

$$\mathbb{E}\max_{x\in\mathcal{X}} f(x) - \mathbb{E}\max_{t=1,2,\ldots,T} f(x_t)$$

Expected cost-adjusted regret

$$\mathbb{E}\max_{x\in\mathcal{X}} f(x) - \mathbb{E}\max_{t=1,2,\ldots,T} f(x_t) + \mathbb{E}\sum_{t=1}^{T} c(x_t)$$

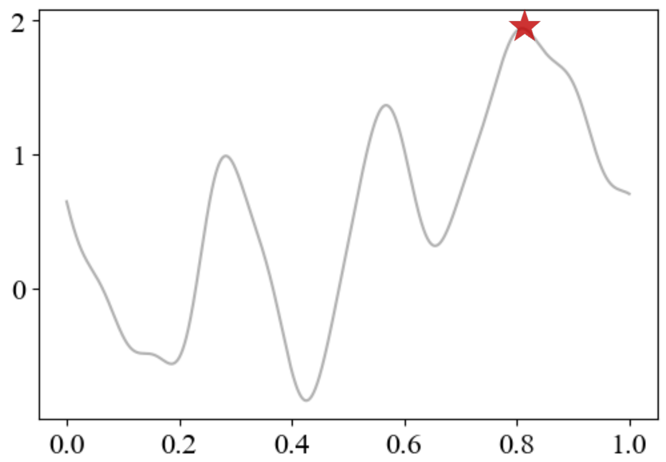
cumulative cost

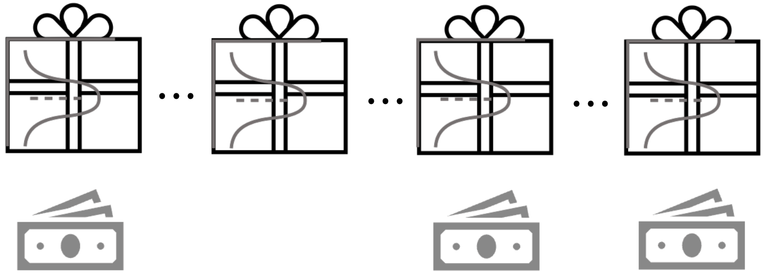
Bayesian Optimization

Pandora's Box

[Weitzman'79]

$f(x)$





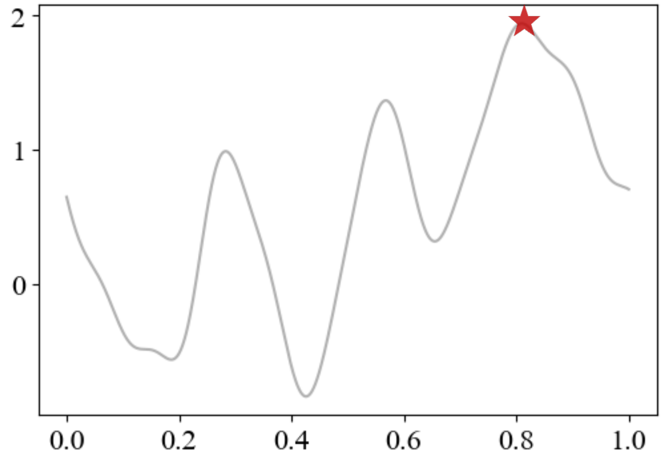
$f(x)$

$c(x)$

Continuous	Discrete
Correlated	Independent
Fixed-budget / Flexible-stopping	Flexible-stopping
Expected (cost-adjusted) regret	Expected cost-adjusted regret

Bayesian Optimization

$f(x)$



Continuous

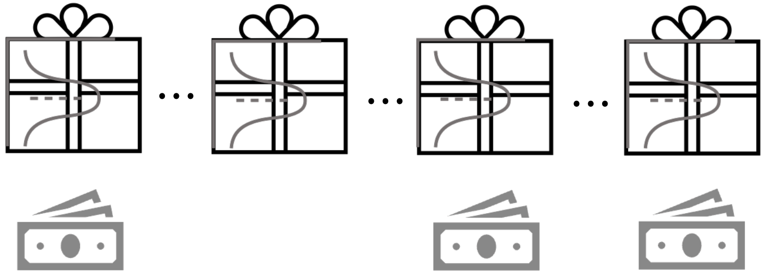
Correlated

Fixed-budget / Flexible-stopping

Expected (cost-adjusted) regret

Pandora's Box

[Weitzman'79]



$f(x)$

$c(x)$

Discrete

Independent

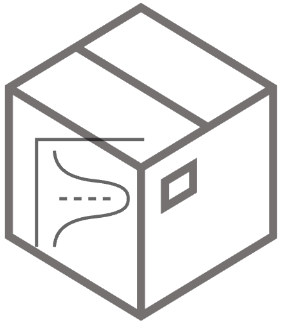
Flexible-stopping

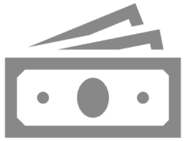
Expected cost-adjusted regret

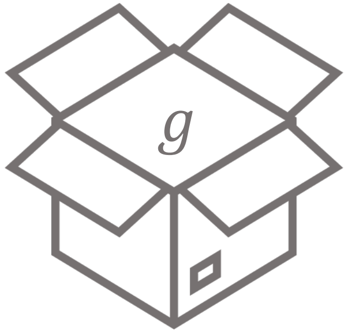
Optimal policy: Gittins index

Optimal Policy: Gittins Index

Step 1: Assign each box a Gittins index (higher is better)

$f(x)$ 

$c(x)$ 

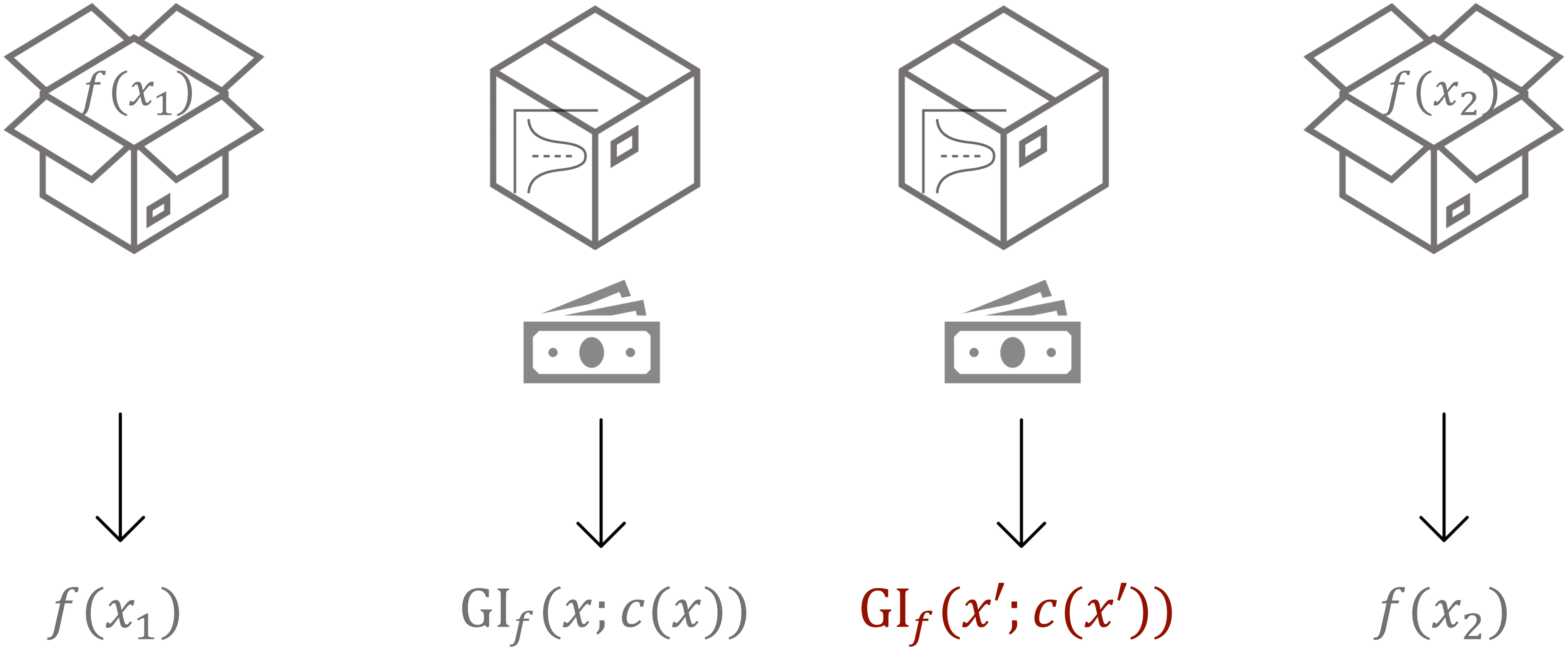


$\downarrow$ $\downarrow$

$\mathrm{GI}_f(x; c(x))$ g

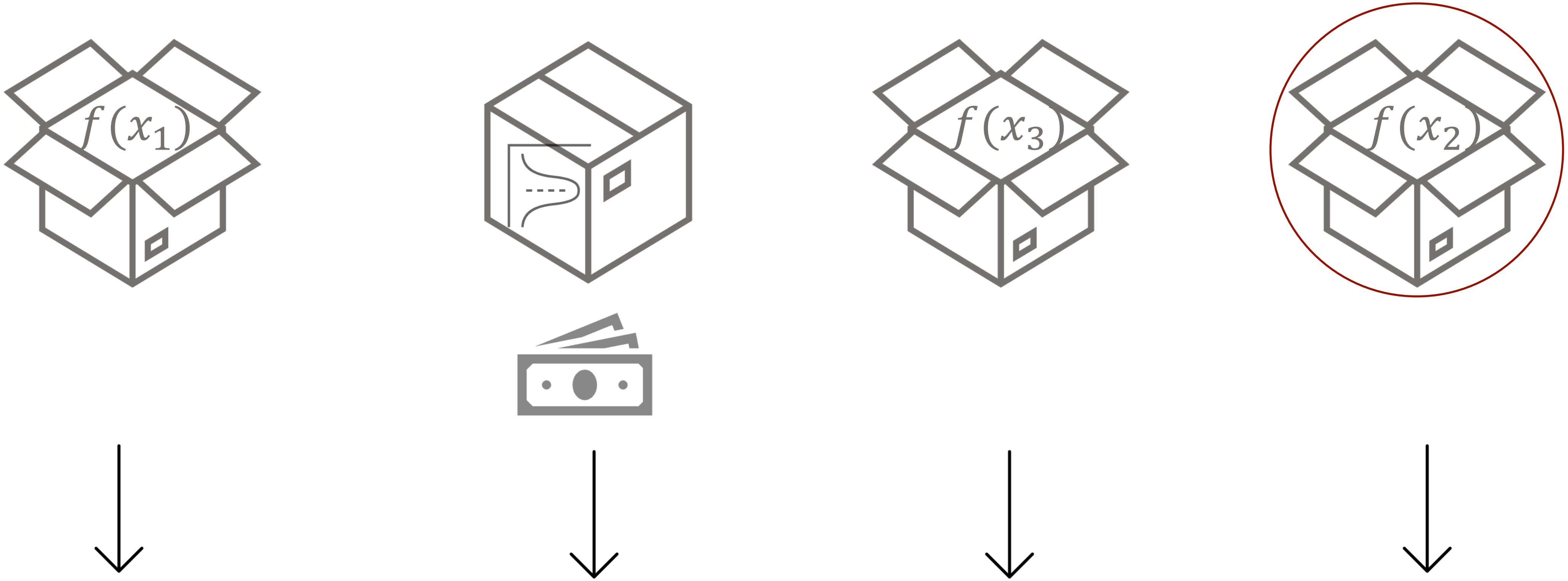
Optimal Policy: Gittins Index

Step 2: Open the box with highest index if it is closed



Optimal Policy: Gittins Index

Step 2’: Select the box with highest index if it is opened and stop



$f(x_1)$ $\quad$ $\mathrm{GI}_f(x; c(x))$ $\quad$ $f(x_3)$ $\quad$ $f(x_2)$

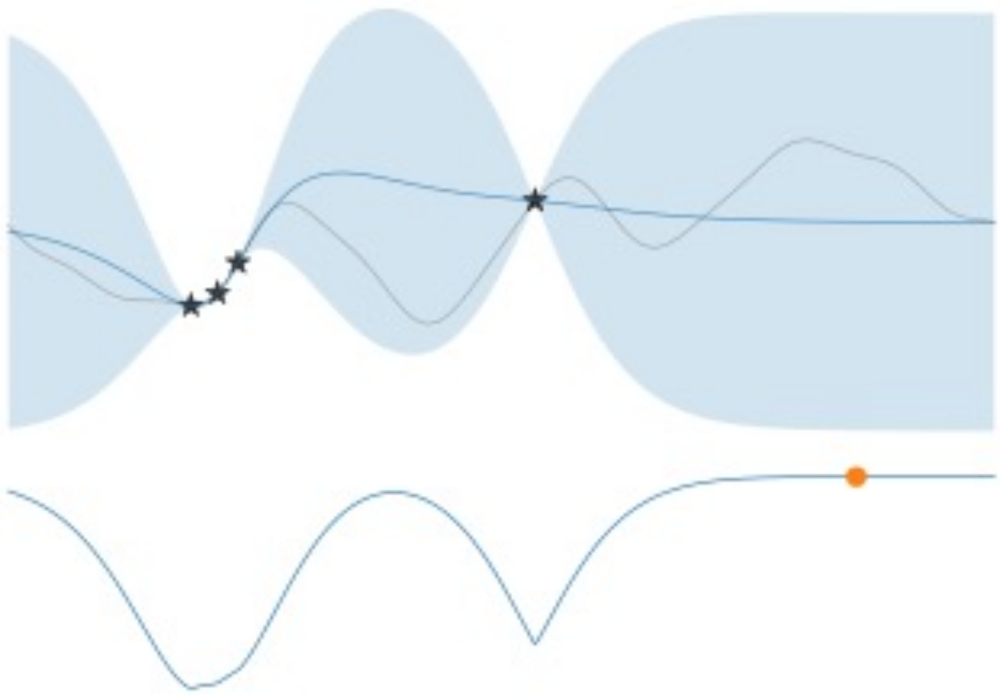
Optimal Policy: Gittins Index

Varying evaluation costs

$$\mathrm{GI}\big(x; c(x)\big)$$

Smart stopping time

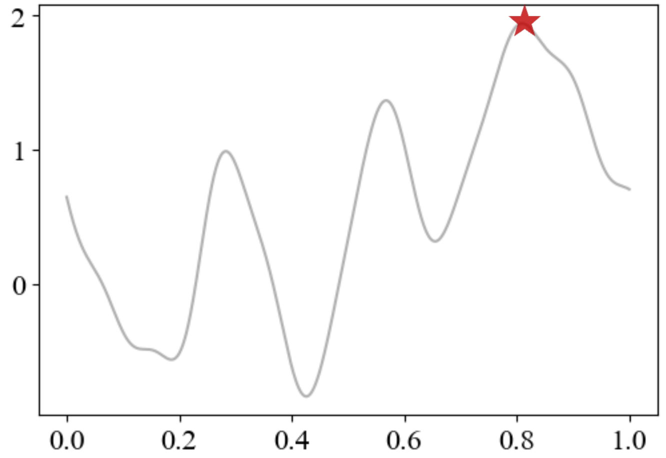
$$\max_x \mathrm{GI}\big(x; c(x)\big) \leq \max_x f(x)$$



Gittins index $\mathrm{GI}(x)$

Bayesian Optimization

$f(x)$



Continuous

Correlated

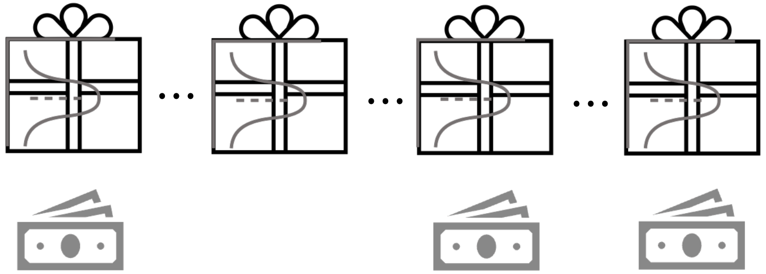
Fixed-budget / Flexible-stopping

Expected (cost-adjusted) regret

Is Gittins index good?

Pandora's Box

[Weitzman'79]



$f(x)$

$c(x)$

Discrete

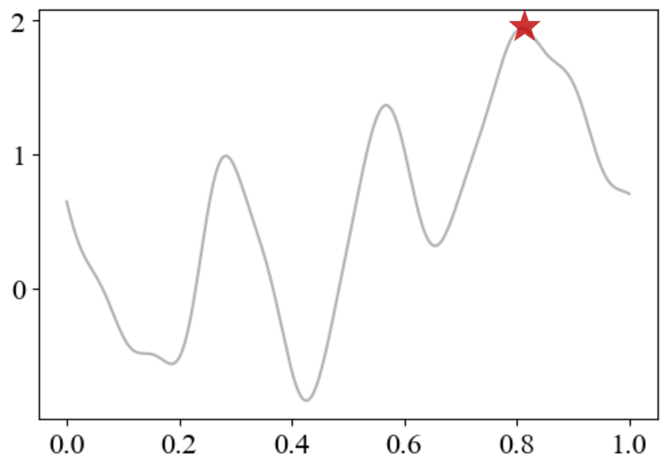
Independent

Flexible-stopping

Expected cost-adjusted regret

Gittins index is optimal

Bayesian Optimization



$f(x)$

Continuous

Correlated

Fixed-budget / Flexible-stopping

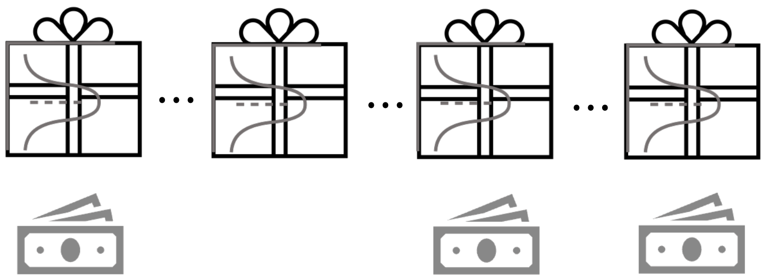
Expected (cost-adjusted) regret

Is Gittins index good?

empirically

Pandora's Box

[Weitzman'79]



$f(x)$

$c(x)$

Discrete

Independent

Flexible-stopping

Expected cost-adjusted regret

Gittins index is optimal

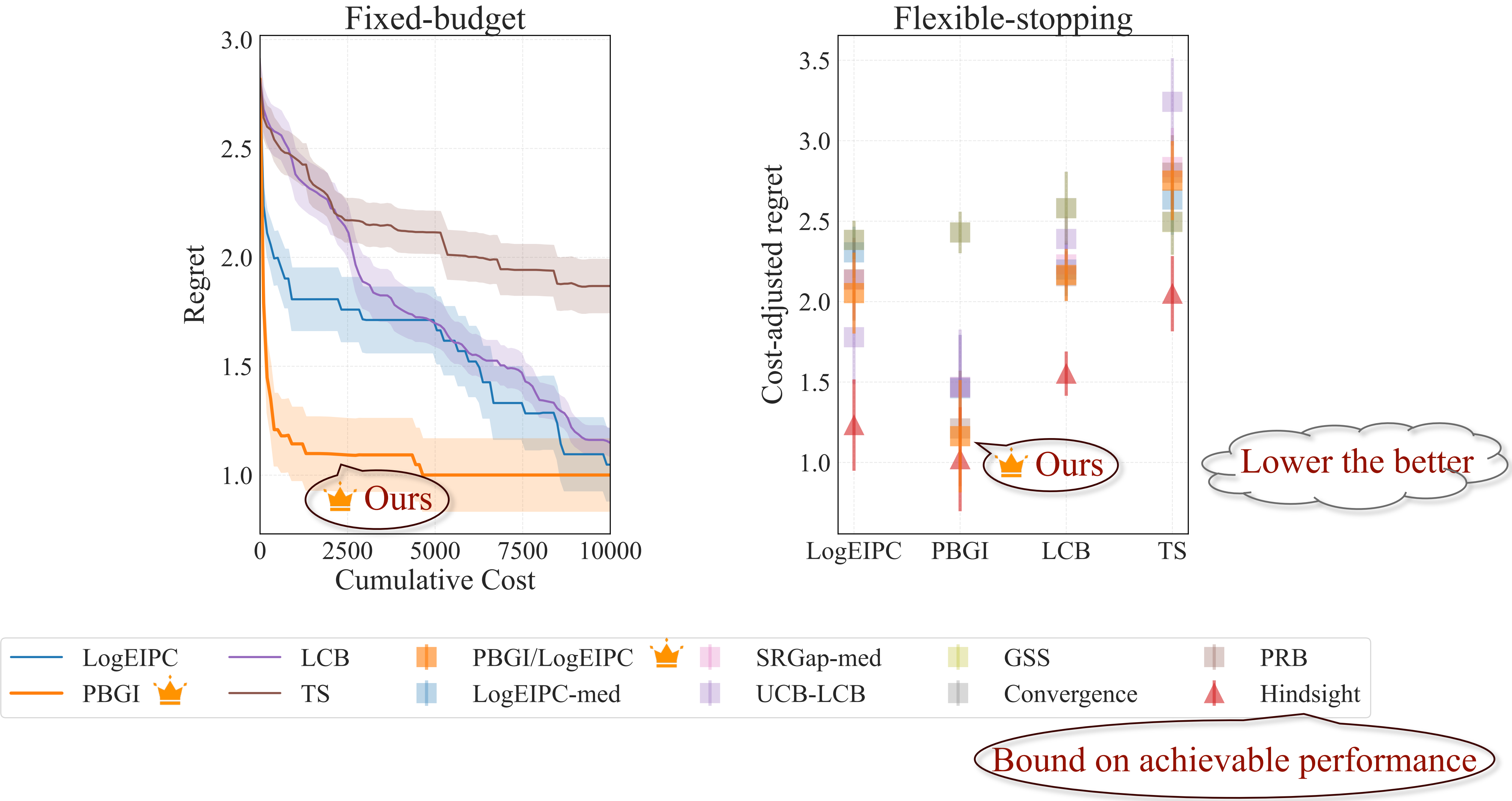
Our Contribution: Gittins Index Principle

- Improvement-based (e.g., LogEIPC)
- Entropy-based
- Confidence bounds (UCB/LCB)
- Thompson sampling (TS)
- Gittins Index (PBGI)

? Why another principle?

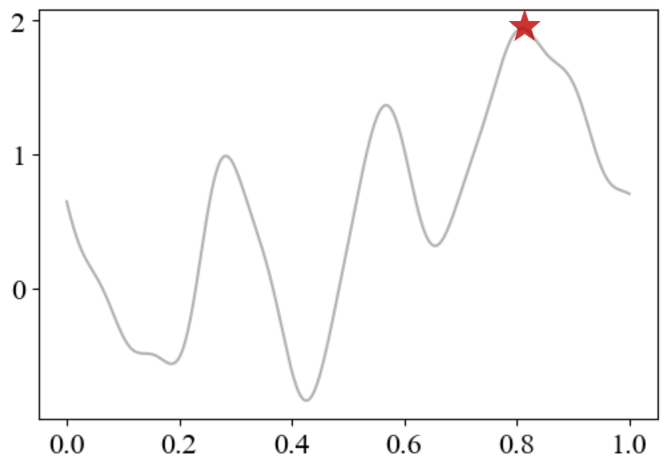
1. Naturally incorporates side info and practical flexibility
2. Performs competitively on benchmarks
3. Comes with theoretical guarantees

Gittins Index vs Baselines on AutoML Benchmark



Bayesian Optimization

$f(x)$



Continuous

Correlated

Fixed-budget / Flexible-stopping

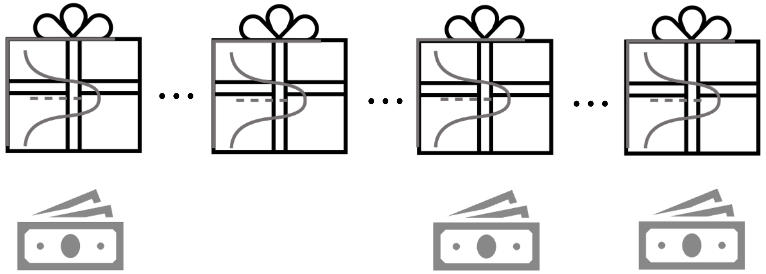
Expected (cost-adjusted) regret

Is Gittins index good?

theoretically

Pandora's Box

[Weitzman'79]



$f(x)$

$c(x)$

Discrete

Independent

Flexible-stopping

Expected cost-adjusted regret

Gittins index is optimal

Our Contribution: Gittins Index Principle

- Improvement-based (e.g., LogEIPC)
- Entropy-based
- Confidence bounds
- Thompson sampling
- Gittins Index

? Why another principle?

1. Naturally incorporates side info and practical flexibility
2. Performs competitively on benchmarks
3. Comes with theoretical guarantees

Theoretical Guarantee and Empirical Validation

Theorem (No worse than stopping-immediately)

$\mathbb{E}[R(\text{ours}; \text{PBGI})] \leq R[\text{stopping immediately}]$

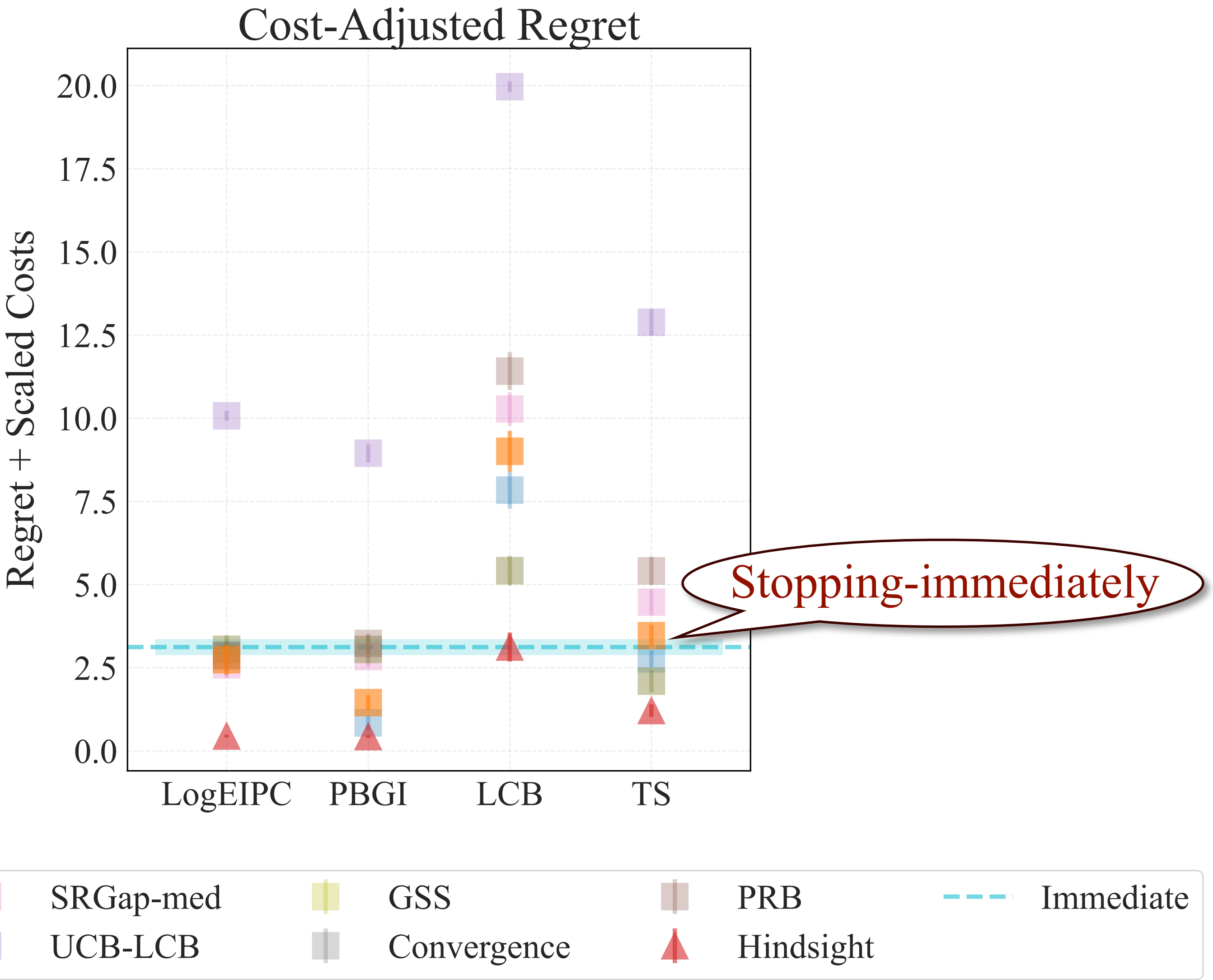
or LogEIPC

cost-adjusted regret

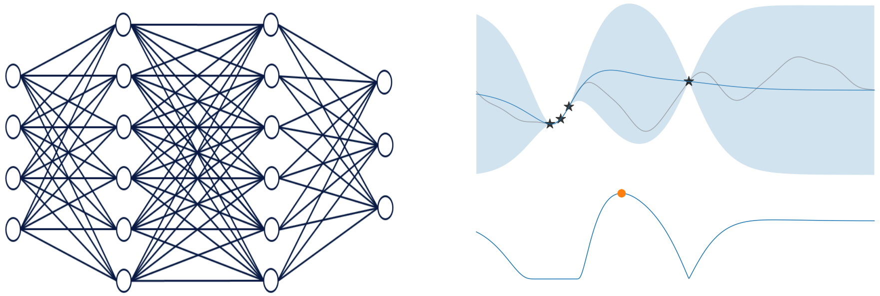
Implication:

- Matches the best achievable performance in the worst case (evaluations are all very costly).

- Avoids over-spending — a property many cost-unaware stopping rules lack.



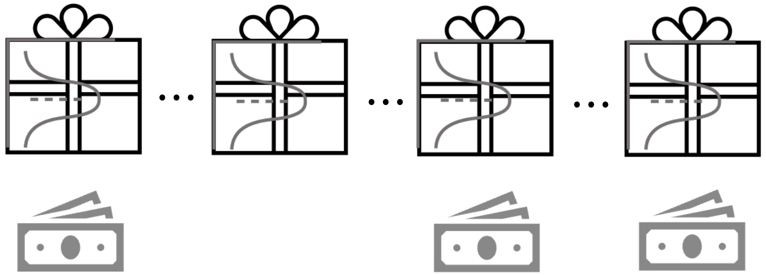
Studied problem



Varying evaluation costs

Adaptive stopping time

Key idea



Link to Pandora's Box problem
& Gittins index theory

Impact

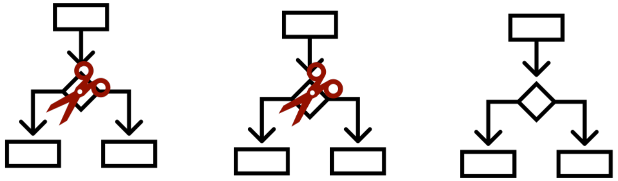
Meta

BoTorch Ax

Competitive empirical performance &
interests from practitioners

"Cost-aware Bayesian Optimization via the Pandora's Box Gittins Index." NeurIPS'24.

Ongoing work



Sharper theoretical guarantees & black-box optimization w/ multi-stage feedback

"Cost-aware Stopping for Bayesian Optimization." Under review.

Find our papers on arXiv!

"Cost-aware Bayesian Optimization via the Pandora's Box Gittins Index.” NeurIPS’24.

"Cost-aware Stopping for Bayesian Optimization.” Under review.