Resilience of Dynamic Routing in the Face of Recurrent and Random Sensing Faults

Qian Xie 谢倩
qx463@nyu.edu

Li Jin 金力
lijin@nyu.edu

Department of Civil & Urban Engineering
C2SMART University Transportation Center

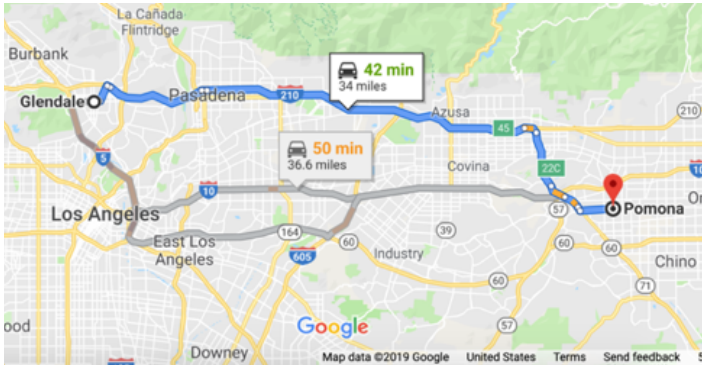
Background

Routing service

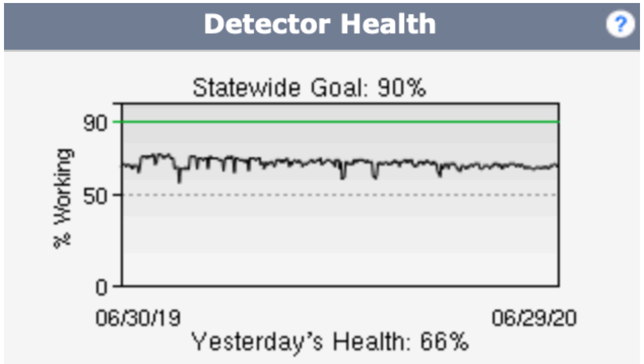
- Provide travelers with dynamic traffic information

Sensing faults

- Cause corrupt or missing data



Recommended multiple parallel routes based on real-time traffic condition



Percentage of working induction loops in PeMS

Literature Review

Research gap

- Most of existing traffic management approaches typically assume complete knowledge of traffic condition.
- A body of work on fault-tolerant control has been developed for electrical/mechanical/aerospace engineering; limited results are available for recurrent and random faults and directly applicable to intelligent transportation system.

Motivation

- Evaluate the impact of faults on fault-unaware routing algorithm and derive practically relevant insights for designing fault-tolerant routing algorithms

Problem Formulation

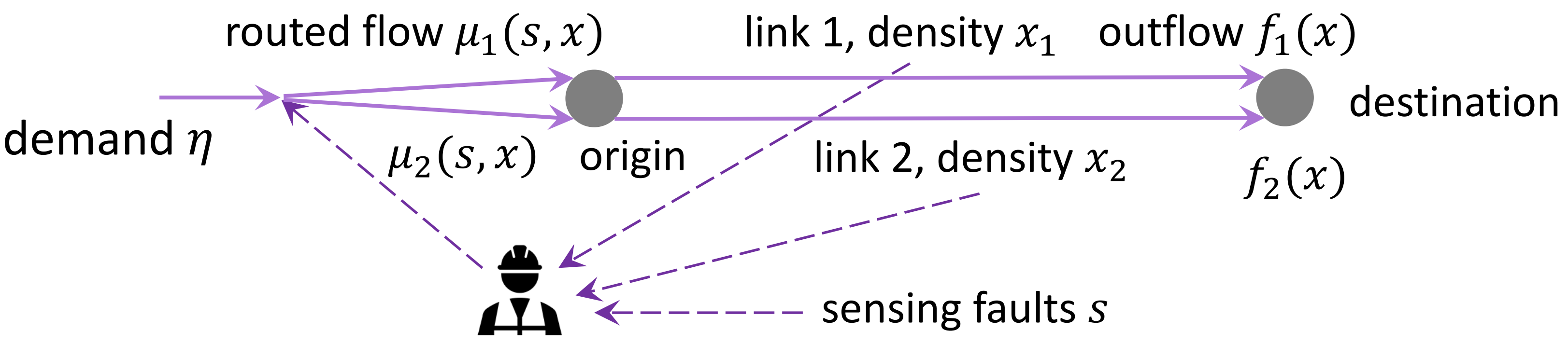
Two-link network

- Bounded & monotonic link flow function

$$f_k(x_k) = F_k(1 - e^{-x_k})$$

- Logit routing based on detected density

$$\mu_k(s, x_k) = \frac{e^{-\beta \hat{x}_k}}{\sum_l e^{-\beta \hat{x}_l}}$$



Problem Formulation (Cont'd)

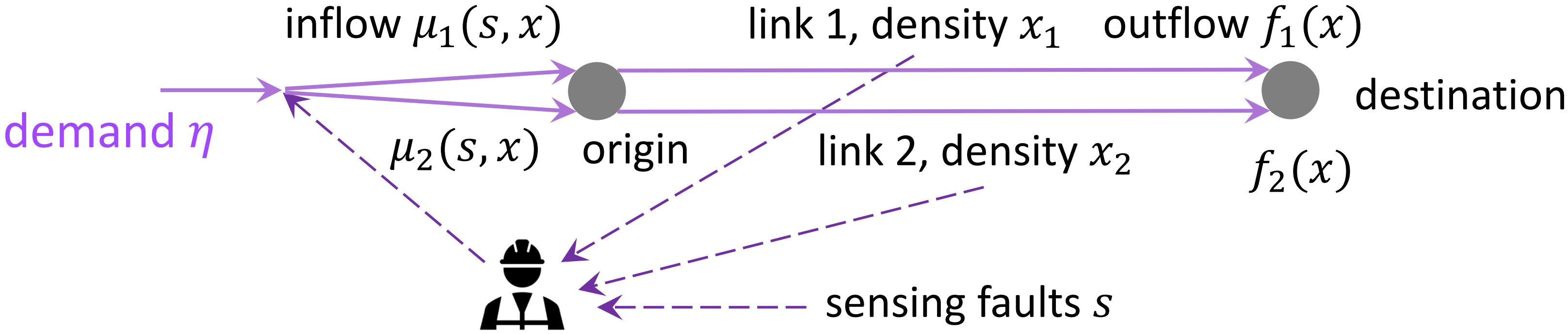
Markovian sensing faults

- Sensing faults induce missing values
- A Markov chain with four observation modes

mode 1: $[x_1, x_2]$, mode 2: $[x_1, 0]$, mode 3: $[0, x_2]$, mode 4: $[0, 0]$

Key question

- Is the system stable given a demand η?



Stability Analysis

The network is stable if

$$\exists Z < \infty, \lim_{t\to\infty} \sup \frac{1}{t}\int_{\tau=0}^{t} \mathrm{E}[X_1(\tau) + X_2(\tau)]\mathrm{d}\tau \leq Z.$$

(bounded time-average link densities)

Necessary condition for stability

$$\eta\left(\frac{1}{e^{-\beta \underline{x}_2}} p_2 + \frac{1}{2} p_4\right) \leq F_1$$

(expected inflow into link 1) (maximum outflow from link 1)

$$\eta\left(\frac{1}{e^{-\beta \underline{x}_1}} p_3 + \frac{1}{2} p_4\right) \leq F_2$$

(expected inflow into link 2) (maximum outflow from link 2)

$$\eta < F_1 + F_2$$

Stability Analysis (Cont'd)

Sufficient condition for stability

$$\exists \theta \in R_{\geq 0}^{2}, \sum_{s=1}^{4} p_s \max_{k \in \{1,2\}} \{\eta \frac{e^{-\beta T_{s,k}(\theta_k)}}{e^{-\beta T_{s,k}(\theta_1)} + e^{-\beta T_{s,k}(\theta_2)}} - F_k(1 - e^{-\theta_k})\} < 0$$

(inflow into link k under mode s) (outflow from link k)

Sketch of proof

- Foster-Lyapunov criterion with the following Lyapunov function

$$V(s, x) = \frac{1}{2}\left((x_1 - \theta_1)_+ + (x_2 - \theta_2)_+\right)^2 + a_s\left((x_1 - \theta_1)_+ + (x_2 - \theta_2)_+\right)$$

where $(x_k - \theta_k)_+ = \max\{x_k - \theta_k, 0\}$.

Resilience Analysis

Guaranteed throughput (resilient score)

- If the two-link network is characterized by a guaranteed throughput η^*, it is stable for any demand $\eta < \eta^*$.

Impact of sensing fault frequency

- Suppose homogeneous capacity $F_1 = F_2$, $\eta^* \geq \frac{1}{1+p_2+p_3}$.
- More sensing faults (higher p_2, p_3) leads to less resilience.

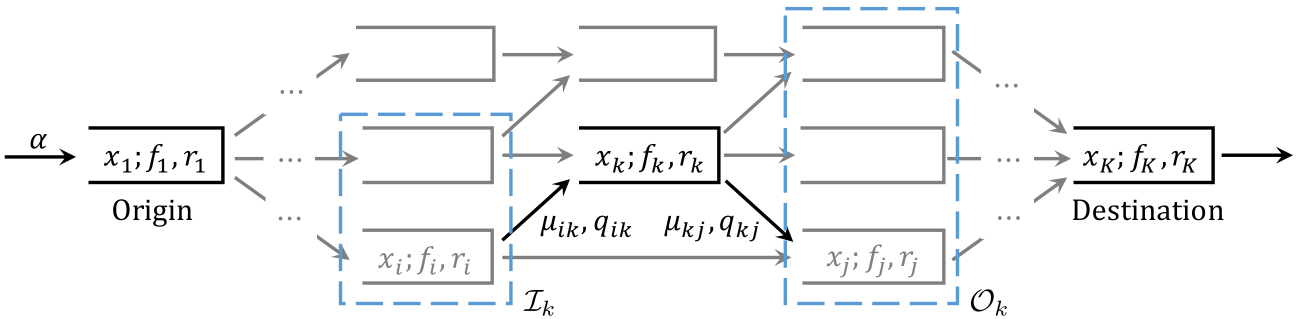
Impact of heterogeneous link capacities

- Suppose $p_2 = p_3$, $\eta^* \geq \min\{\frac{1-(F_1-F_2)}{1-p_1}, \frac{1-p_4(F_1-F_2)}{1+2p_2}\}$
- Larger difference of capacities $(F_1 - F_2)$ leads to less resilience.

General Case*

General network

- Single-origin-single-destination, acyclic network



Cyber-physical disruptions

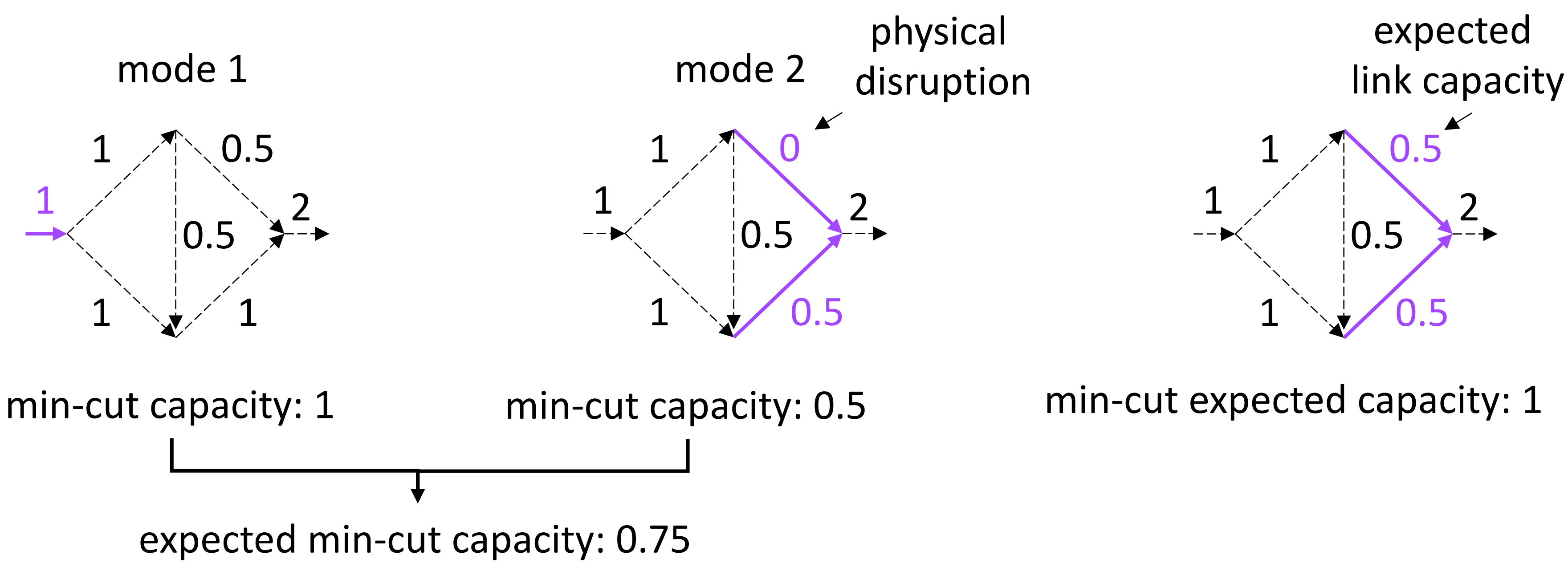
- Physical disruptions, such as accidents, undermine link flow.

*Tang, Y., Jin, L. (2020). Analysis and Control of Dynamic Flow Networks Subject to Stochastic Cyber-Physical Disruptions, submitted to Automatica.

General Case (Cont'd)

Network capacity under cyber-physical disruptions

- Expected min-cut capacity
- Min-cut expected capacity



Two Special Cases

Disruption-awareness

- Suppose real-time disruption modes are available, there exists a mode-dependent control such that the network attains expected min-cut capacity.

Infinite link storage

- Suppose links have infinite storage, there exists an open-loop control such that the network attains min-cut expected capacity.

Sketch of proof

- Foster-Lyapunov criterion

General Control

Limitations of the two special cases

- Hard to real-time detect disruption modes;
- Few links have infinite storage.

Solutions

- Design feedback control with guaranteed throughputs using the knowledge of disruptions:

Suppose control law μ satisfies

$$1)\ \bar{x}_k^{\mu} < \infty, \forall k \neq 1 \text{ and } 2)\ \sum_i \mu_{ik}(s,x) \leq r_k(s,x),$$

the network can attain a lower-bounded guaranteed throughput.

Summary

- Propose a novel model that synthesizes traffic flow dynamics and stochastic sensing faults
- Establish a stability criterion to check whether the system is stable
- Evaluate the impact of faults on traditional routing algorithm in terms of guaranteed throughput
- Extend the methodology for a rather general network subject to cyber-physical disruptions